LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - MATHEMATICS

THIRD SEMESTER – NOVEMBER 2013

MT 3811 - COMPLEX ANALYSIS

Date : 08/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

Answer all the questions.

1. a) Let G be a region. Show that any analytic function $f: G \to \mathbb{C}$ such that $|f(z)| \leq |f(a)|\ \forall\, z \in G$ is constant.

OR

b) State and prove Fundamental theorem of Algebra. (5)

c) State and prove Goursat's theorem.

OR

d) State and prove homotopic version of Cauchy's theorem. (15)

2. a) Prove that any differentiable function f on $[a, b]$ is convex if and only if f' is increasing.

OR

b) State and prove Schwarz lemma. (5)

c) State and prove Arzela Ascoli theorem.

OR

d) State and prove Riemann mapping theorem. (15)

3. a) Let $Re z_n > 0$, for all $n \geq 1$. Then prove that $\prod_{k=1}^{\infty} z_k$ converges to a complex number different from zero if and only if $\sum_{k=1}^{\infty} log z_k$ converges.

OR

b) Show that $\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2}\right)$. (5)

c) (i) If $Re\, z > 0$ then prove that $\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1}\, dt$.

(ii) State and prove Weierstrass factorization theorem. (7+8)

OR

d) (i) State and prove Bohr-Mollerup theorem.

(ii) State and prove Euler's theorem (8+7)

4. a) State and prove Jensen's formula.

OR

b) Let f be an entire function of finite order. Then prove that f assumes each complex number with one possible exception. (5)

c) State and prove Mittag-Leffler's theorem.

OR

d) If f is an entire function of finite order λ, then prove that f has finite genus $\mu \leq \lambda$.

(15)

5. a) Prove that an elliptic function without poles is a constant.

OR

b) Prove that $\wp(z)$ is doubly periodic. (5)

c) (i) Prove that a discrete module consists either of zero alone, of the integral multiples nw of a single complex number $w \neq 0$ or of all linear combinations $n_1 w_1 + n_2 w_2$ with integral coefficients of two numbers w_1, w_2 with non real ratio $\frac{w_2}{w_1}$.

(ii) Show that $\begin{vmatrix} \wp(z) & \wp'(z) & 1 \\ \wp(u) & \wp'(u) & 1 \\ \wp(u+z) & \wp'(u+z) & 1 \end{vmatrix} = 0$ (8+7)

OR

d) (i) Derive Legendre's relation.

(ii) State and prove the addition theorem for the Weierstrass $\wp$ function. (7+8)